Influence of Surface Tension on Jet-Stripped Continuous Coating of Sheet Materials

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INTRODUCTION

The use of a stripping jet or "air knife" in continuous coating processes has been investigated by Thornton and Graff (1976) and by Tuck (1983). In these studies, the jet is represented as a given distribution of pressure within the layer of liquid material. However, the external pressure of the jet must communicate itself to the coating layer across its free surface, and if that free surface possesses curvature and nonzero surface tension, the internal pressure will differ from that in the jet.

We use lubrication-type simplifications to reduce the general unsteady problem with prescribed jet pressure $P(y,t)$ and shear $T(y,t)$ to that of solving a partial differential equation for the layer thickness $h(y,t)$ as a function of height y and time t. This derivation reveals that surface-tension effects are important if the capillary number (i.e., nondimensional inverse surface tension) is comparable to or smaller than the cube of the slope or thickness/height scale of the layer.

The influence of surface tension is then explored numerically, by solving the ordinary differential equation that describes the steady-state thickness $h(y)$. Surface tension has the effect of inhibiting the stripping tendency of the jet. That is, the reason for use of the jet is to obtain thinner coatings than would arise from pure draining under gravity. Since this introduces curvature into the free surface of the coating layer, surface tension will resist it, and the influence of the jet will be less than if there was no surface tension. The present numerical results enable quantitative estimation of the magnitude of this effect.

LUBRICATION APPROXIMATION

Assuming two-dimensional flow of an incompressible Newtonian viscous fluid, Figure 1, the exact problem requires us to solve the Navier-Stokes equations with boundary conditions

$$u = 0, \qquad v = V \qquad \text{on } x = 0. \tag{1}$$

and

$$u = h_t + vh_y \qquad \text{on } x = h, \tag{2}$$

where the free boundary has equation $x = h(y,t)$. We suppose that there is a free-surface pressure $P(y,t)$ and tangential stress $T(y,t)$, and that the interface possesses surface tension σ. Thus, the dynamic boundary conditions require that on $x = h$,

$$\tau_\perp = -P + \sigma h_{yy}(1 + h_y{}^2)^{-3/2} \tag{3}$$

and

$$\tau_\parallel = T \tag{4}$$

where $\tau_\perp$, $\tau_\parallel$, are normal and tangential stresses in the fluid. We envisage P and T as given quantities, due to the action of an ex-

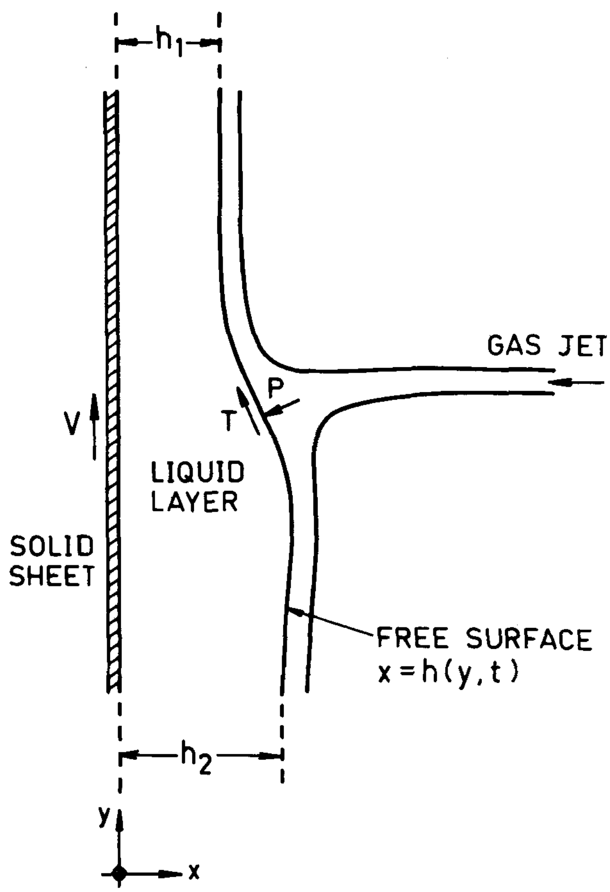

Figure 1. Flow and coordinate system.

Please see letters to the editor on page 879 for further discussion of this paper.

ternally-applied gas jet, although this is not a necessary assumption.

We now suppose that this boundary has a small $0(\epsilon)$ *slope* to the vertical. That is, if L denotes a length scale for changes in the y-direction, prescribed for example by the y-wise scale of the pressure P, we assume that any layer of interest has a thickness much less than L, with $h/L = 0(\epsilon)$, $\epsilon \ll 1$. This is the situation that applies in lubrication theory (Cameron, 1966), and allows asymptotic simplifications in the limit as $\epsilon \to 0$. In order that gravity and the jet both have an effect, we must demand that the hydrostatic pressure $\rho g L$, the jet pressure P, and the fluid pressure p all have the lubrication pressure scale, namely

$$\rho g L, P, p = 0(\mu V L^{-1} \epsilon^{-2}), \tag{5}$$

where μ is the viscosity. We must also demand that the Reynolds' number be not too large, specifically that

$$\rho V L/\mu \ll \epsilon^{-2}. \tag{6}$$

The Navier-Stokes equation then simplify (cf. Atherton and Homsy, 1976) to

$$p_x = 0 \tag{7}$$

$$v_{xx} = \frac{\rho g + p_y}{\mu} \tag{8}$$

which are to be satisfied in $0 < x < h$, subject to Eq. 1 on $x = 0$ and Eq. 2,

$$p = P - \sigma h_{yy}, \tag{9}$$

and

$$v_x = T/\mu \tag{10}$$

on $x = h$.

Note that Eq. 9 is consistent in order of magnitude only if $\sigma = 0(\mu V \epsilon^{-3})$. If Ca denotes the capillary number

$$Ca = \mu V/\sigma \tag{11}$$

then we need

$$Ca = 0(\epsilon^3). \tag{12}$$

If $Ca \gg 0(\epsilon^3)$, surface tension is negligible. Similarly, Eq. 10 is consistent in order of magnitude only if $T = 0(\mu V L^{-1} \epsilon^{-1})$ i.e.

$$T/P = 0(\epsilon). \tag{13}$$

If $T/P \ll 0(\epsilon)$, shear forces due to the jet are negligible.

SOLUTION OF FLOW EQUATIONS

In view of Eq. 7 $p = p(y,t)$. and hence by Eq. 9,

$$p(y,t) = P(y,t) - \sigma h_{yy}(y,t) \tag{14}$$

is known through the flow field, once $h(y,t)$ is known. The solution of Eq. 8 subject to Eqs. 1 and 10 is

$$v = V + \frac{\rho g + P_y - \sigma h_{yyy}}{\mu} (\tfrac{1}{2} x^2 - hx) + \frac{Tx}{\mu} \cdot \tag{15}$$

The corresponding x-wise velocity is obtained by solving the continuity equation subject to Eq. 1, with the result

$$u = h_y \frac{\rho g + P_y - \sigma h_{yyy}}{2\mu} x^2 - \frac{P_{yy} - \sigma h_{yyyy}}{\mu} \times \left(\frac{1}{6} x^3 - \frac{1}{2} h x^2 \right) - \frac{T_y x^2}{2\mu} \cdot \tag{16}$$

Finally, the kinematic boundary condition (Eq. 2), gives

$$h_t + c h_y = f - \frac{\sigma}{3\mu} (h^3 h_{yyy})_y \tag{17}$$

where

$$c = V + \frac{T}{\mu} h - \frac{\rho g + P_y}{2\mu} h^2 \tag{18}$$

and

$$f = \frac{h^3}{3\mu} P_{yy} - \frac{h^2}{2\mu} T_y. \tag{19}$$

In general, Eq. 17 is a partial differential equation to determine $h = h(y,t)$. One may hope to solve it, for any initial profile $h(y,0)$. If $\sigma = 0$, i.e., if there is no surface tension, or more accurately in view of Eq. 12, if $Ca \gg \epsilon^3$, Eq. 17 is a first-order equation and can be solved as in Homsy and Geyling (1977) or Tuck (1983).

In the special case of steady flow in which no flow variable depends upon time t, Eq. 17 integrates once to give (with primes for d/dy)

$$Q = Vh + \frac{T}{2\mu} h^2 - \frac{\rho g + P' - \sigma h'''}{3\mu} h^3 \tag{20}$$

where Q is a constant, physically identifiable as the net flux

$$Q = \int_0^h v\, dx. \tag{21}$$

Thus, constancy of Q simply reflects conservation of mass for steady flow.

EFFECT OF SURFACE TENSION ON PRESSURE STRIPPING

In order to concentrate attention on the effect of surface tension, we now set $T = 0$ in Eq. 20, giving

$$Q = Vh - \frac{\rho g + P' - \sigma h'''}{3\mu} h^3 \tag{22}$$

That is, we assume that the jet's stripping action is dominated by its pressure P, with $T/P \ll 0(\epsilon)$, as in Eq. 13. A complementary discussion of the problem with surface shear T included, but surface tension σ neglected, is provided by Ellen and Tu (1983).

When $\sigma \neq 0$, (22) is a third-order ordinary *differential* equation for $h = h(x)$, and generalises one given in Levich (1962) for $P(y) \equiv 0$. On the other hand, if $\sigma = 0$, Eq. 22 reduces to a cubic *algebraic* equation for h. If, further, $P(y) \equiv 0$, this cubic equation is independent of y, namely

$$Q = Vh - \frac{\rho g}{3\mu} h^3, \tag{23}$$

and thus its solutions represent *uniform* coating thickness, $h =$ constant.

As a function of h, the righthand side of Eq. 23 possesses a maximum, of value

$$Q_m = \frac{2}{3} V h_m, \tag{24}$$

at $h = h_m$, where

$$h_m = (V\nu/g)^{1/2}. \tag{25}$$

So long as $0 < Q < Q_m$, there exist *two* positive solutions $h = h_s$ and $h = h_L$ of the cubic equation (Eq. 23) satisfying

$$0 < h_s < h_m < h_L. \tag{26}$$

The hypothesis of Deryaguin (1945) and Hrbek (1961) is that, in the absence of the jet, the actual thickness of the coating is $h = h_m$ as given by Eq. 25, namely that value that maximises the flux. The purpose of the jet is to *reduce* the actual coated thickness at $y = +\infty$ below this value $h = h_m$, i.e., to the value $h = h_s$, for some $Q < Q_m$.

Any such jet will have a finite range of effect, and in particular, far above or below the position where it acts, we expect that $P \to 0$. Hence solutions of Eq. 22 will approach solutions of Eq. 23, for some Q, and h will become asymptotically constant. We therefore impose vanishing slope and curvature conditions, i.e. set

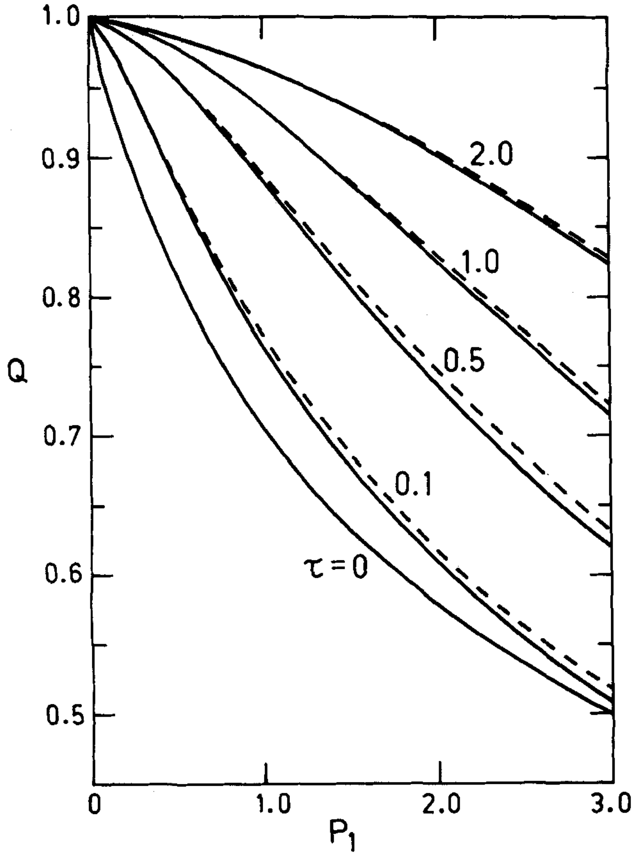

Figure 2. Scaled coating flux Q vs. maximum pressure gradient P_1, for various values of the surface-tension parameter τ. The solid curves are for a piecewise quadratic pressure gradient, and the dashed curves for an exponential pressure distribution.

$$h', h'' \to 0 \qquad \text{as } y \to \pm\infty. \tag{27}$$

The boundary conditions (Eq. 27) do not in themselves prescribe which of the values $h = h_s$ or $h = h_L$ will be approached. However, Eq. 22 is a nonlinear equation that will be solved by iteration. We do not expect that the system (Eqs. 22, 27) has a unique solution, and the solution that one actually achieves in any successfully completed iterative procedure will depend on the nature of the starting guess. We therefore "seed" our solution, by starting with a guessed profile $h(y)$ having $h > h_m$ for $y < 0$, and $h < h_m$ for $y > 0$. This causes convergence toward a solution with $h \to h_L$ as $y \to -\infty$, and $h \to h_s$ as $y \to +\infty$.

Equation 22 can be written in nondimensional form as follows. If $h = h_m h^*$, $Q = Q_m Q^*$, $P = \rho g P^*$ and $y = Ly^*$, where h_m and Q_m are as defined by Eqs. 25 and 24, then (omitting stars),

$$1 + P'(y) - \tau h'''(y) = \frac{3h - 2Q}{h^3} \tag{28}$$

where

$$\tau = \sigma h_m/(\rho g L^3) \tag{29}$$

i.e.

$$\tau^2 = Ca.Bo^3 = \sigma^2 \mu V \rho^{-3} g^{-3} L^{-6} \tag{30}$$

where Ca is the capillary number as defined earlier, and

$$Bo = \sigma/(\rho g L^2) \tag{31}$$

is a Bond number. In fact, it is the parameter τ that truly measures the influence of surface tension in the present problem; if τ is small, surface tension has a negligible effect. A singular perturbation treatment of the limit as $\tau \to 0$ of an equation similar to Eq. 28 has recently been provided by Howes (1983).

We use the following direct numerical method to solve Eq. 28 subject to Eq. 27. Let Δy be a suitably small interval, and write $h_j = h(y_0 + j\Delta y)$, $j = 1,2,\ldots N-1$, where $y_0 \ll 0$ and $y_0 + N\Delta y \gg 0$. We satisfy Eq. 27 by introducing four extra points h_{-1}, h_0, h_N, h_{N+1} such that $h_{-1} = h_0 = h_1$ and $h_{N+1} = h_N = h_{N-1}$. Now, on this extended mesh, we use the second-order accurate estimate

$$h'''(y_0 + (j - \tfrac{1}{2}\Delta y) = \frac{h_{j+1} - h_{j-2} + 3h_j - 3h_{j-1}}{\Delta y^3} \tag{32}$$

for the third derivative in Eq. 28 and force Eq. 28 to hold at $y = y_0 + (j - \tfrac{1}{2})\Delta y$, $j = 1,2,3,\ldots N$, by linearly interpolating the right-hand side. This gives N nonlinear algebraic equations, that we solve by Newton iteration. There are a total of N unknowns, namely the $N-1$ values of $h = h_j$, $j = 1,2,\ldots,N-1$, and the unknown flux Q.

In fact, it is Q itself that is the primary output of interest, since this quantity measures the final (solidified) coating thickness. Figure 2 shows Q vs. $P_1 = \max P'(y)$, for various values of τ. The curve for $\tau = 0$ is that first given by Thornton and Graff (1976), namely

$$Q = (1 + P_1)^{-1/2} \tag{33}$$

Notably, this zero-surface-tension result depends only on the nondimensional maximum pressure gradient P_1, and is quite independent of the shape of the $P'(y)$ curve.

If $\tau > 0$, the shape of the $P'(y)$ curve does influence the value of Q, and Figure 2 includes two different examples. The solid curves are for a piecewise-quadratic pressure gradient, i.e.

$$P'(y)/P_1 = \begin{cases} 0 & y < -1, \\ -4y - 4y^2, & -1 < y < 0, \\ -4y + 4y^2 & 0 < y < 1, \\ 0 & y > 1, \end{cases} \tag{34}$$

with peaks at $y = \pm\tfrac{1}{2}$. The dashed curves are for a Gaussian-type pressure distribution, whose gradient

$$P'(y)/P_1 = -\frac{3}{2} y \exp(1 - 9ey^2/8) \tag{35}$$

has peaks at $y = \pm\tfrac{2}{3}e^{1/2} \simeq \pm 0.4$. The normalization is such that in both cases the maximum nondimensional pressure gradient is P_1, and the maximum nondimensional pressure is $\tfrac{2}{3}P_1$. Thus, the length scale L used in the definition 29 of τ has been taken as $1\tfrac{1}{2}$ times the ratio between the actual maximum pressure and the actual maximum pressure gradient.

As τ increases above the zero value where Eq. 33 applies, a stronger jet (larger P_1) is needed to make the same stripping effect, since surface tension tends to keep the free surface plane. At the same time, different shapes of jet pressure distribution are in principle capable of yielding different stripping properties. However, at least when the length scale L is defined as above in such a way as to keep constant both the maximum pressure gradient and the maximum pressure, there is only a very small difference between the results using Eqs. 34 and 35, over the complete range of values of τ used.

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NOTATION

Bo	$= \sigma/(\rho g L^2)$, Bond number, dimensionless
c	= wave speed, m·s^{-1}
Ca	$= \mu V/\sigma$, capillary number, dimensionless
f	= forcing term in equation for h, m·s^{-1}
g	= acceleration of gravity, m·s^{-2}

h = layer thickness, m
h_1,h_2 = values of h for $Q < Q_m$, m
h_m = maximum value of h, m
h_j = discrete values of h, m
L = length scale in y-direction, m
N = number of discrete points, dimensionless
p = fluid pressure, $N \cdot m^{-2}$
P = jet pressure, $N \cdot m^{-2}$
P_1 = scaled maximum pressure gradient, dimensionless
Q = volume flux per unit span, $m^2 \cdot s^{-1}$
Q_m = maximum value of Q, $m^2 \cdot s^{-1}$
t = time, s
T = tangential stress in jet, $N \cdot m^{-2}$
u = horizontal fluid velocity, $m \cdot s^{-1}$
v = vertical fluid velocity, $m \cdot s^{-1}$
V = upward speed of sheet, $m \cdot s^{-1}$
x = horizontal coordinate, m
y = vertical coordinates, m

Greek Letters

ϵ = measure of surface slope h/L, dimensionless
μ = fluid viscosity, $N \cdot m^{-2} \cdot s^{-1}$
ν = μ/ρ
ρ = fluid density, $kg \cdot m^{-3}$
σ = surface tension coefficient, $N \cdot m^{-1}$
$\tau_\perp$ = normal stress, $N \cdot m^{-2}$
$\tau_\parallel$ = tangential stress, $N \cdot m^{-2}$
τ = $\sigma \mu^{1/2} V^{1/2} \rho^{-3/2} g^{-3/2} L^{-3}$, dimensionless

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Plasma Decomposition of Carbon Dioxide

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The energy efficiency of plasmachemical processes seems to be the most important factor which limits their practical application. Most of plasmachemical reactions under investigation are carried out far from optimum conditions, as their mechanism is often not known. Hence there is the necessity of intensive research into the energy efficiency of plasmachemical reactors and into modelling studies.

The objective of the present work was to model some aspects of the plasma decomposition of carbon dioxide based upon the experimental measurements of fundamental process parameters.

Because of the very high dissociation energy for carbon monoxide molecule, CO_2 decomposition at plasma temperatures occurs primarily according to simple equation

$$CO_2 \rightarrow CO + 0.5\,O_2 \quad (1)$$

Nishimura et al. (1974) observed that even under extreme conditions (reaction temperature as high as 7,500 K) decomposition of CO did not exceed 35%. The simplicity of main reaction, as a model for endothermic homophase processes, is not the only reason for the numerous studies on plasma CO_2 decomposition. There are also some interesting possible uses of the reaction products, mainly as energy sources. Carbon monoxide can be used as an effective reducing agent in metallurgy, an alternative reagent in catalytic processes of organic chemistry and, in the water conversion reaction as a hydrogen source. The results of studies on plasma CO_2 conversion can be useful in formulating solution for high-temperature combustion problems and in species lifetime predictions for high-power lasers. Finally, the problems of rational CO_2 utilization begins to be important for environmental protection reasons (Wigley et al., 1980).

Butylkin et al. (1979) applied numerical techniques to solution of quenching and kinetics problems in high-temperature CO_2 decomposition process and to determine energy consumptions for oxygen production at different quenching rates. Carbon dioxide decomposition has been studied both in nonequilibrium (Brown and Bell, 1974; Legasov et al., 1977) and equilibrium plasmas. Fractional conversions as high as 70% have been obtained in r.f. argon plasma at temperatures exceeding 6,000 K under atmospheric pressure (Nishimura and Takenouchi, 1976). In contradiction to their previous work (Nishimura et al., 1974), the authors did not confirm the decomposition of the produced carbon monoxide. It was stressed, however, that equilibrium was not reached. Blanchet et al. (1969) reported that only 5% conversion was achieved in a d.c. argon plasma jet. Szymański and Huczko (1978) examined the influence of process parameters on the reaction. The total conversion of CO_2 as high as 60% was attained in an argon plasma. Extensive studies on the reaction run in helium plasma have been carried out by Charette and Parent (1973). The authors

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